Preventing Prevention

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Abstract

Most conceptions of electoral accountability rely upon voters’ abilities to process and act upon information about the performance of incumbent politicians in a rational manner. Yet recent empirical evidence suggest that voters do a particularly poor job of this in the context of disasters, failing to reward incumbents for beneficial actions and punishing incumbents for events arguably beyond their control. We develop a simple model of elections in the context of disasters, which shows that (at least some of) these empirical regularities are consistent with rational choice by voters uncertain about politicians’ motivations. In many cases, the unique equilibrium of the model entails suboptimal disaster prevention policy by all incumbents, even though voters are rational and make the best possible use of their blunt tool of political accountability.

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Electoral accountability is a foundational justification for (indirect) democratic governance. Accordingly, it is unsurprising that the determinants and measurement of its effectiveness have attracted significant theoretical and empirical attention (see Ashworth (2012) for a recent review). The classical portrait of a well-functioning democracy is that incumbents, seeking reelection, will make decisions in line with their constituents’ interests in order to secure those constituents’ votes in the next election.

Recent empirical evidence that calls into question the ability (or, perhaps, willingness) of voters to effectively perform their role in holding elected representatives accountable to the voters’ interests. For example, Healy and Malhotra (2009) show that voters do not electorally reward disaster prevention spending, but do reward disaster relief spending. Yet a dollar of prevention spending is an order of magnitude more beneficial than a dollar of relief spending. Healy and Malhotra suggest that voter myopia is a likely cause of this misalignment of electoral incentives and public welfare.

Such a conclusion is obviously troubling in at least two ways. First, it raises serious concerns about whether electoral democracy can (or at least does) yield policy choices that are representative of the citizens’ interests. Second, it leads to doubts about the ability of voters to make make choices that are representative of their own interests. The main point of this article is that this second conclusion, even if one sets aside measurement questions, rests on less than firm theoretical grounds. In other words, we provide a simple model of public policymaking in the shadow of electoral accountability enforced by rational voters that predicts perverse behavior by reelection-seeking incumbents. Accordingly, our argument at least mitigates worrisome conclusions one might draw from seemingly perverse voter behavior about voter rationality. Instead, our theory suggests that the empirical observations might very well mirror perverse incentive problems common to many principal-agent situations.

In a nutshell, competitive elections represent one, albeit partial, way to
align incumbents’ incentives with the wishes of voters. The partial nature of this aligning force is due to various realities of both governance and elections, the most important of which is the conflict between the optimal solutions to the moral hazard and adverse selection dimensions inherent to the agency problem facing a voter represented by a reelection-seeking official. Moral hazard describes the lack of direct (i.e., intrinsic) incentive for any incumbent to work on behalf (i.e., represent the interests) of the voter.\(^1\)

On the other hand, adverse selection problems are based on differential incentives among incumbents. In simple terms, adverse selection is at play if some incumbents are “bad apples” and others are “better apples” from the voter’s perspective. Our argument in this article boils down to considering the possibility that adverse selection—and the incentives facing both incumbents and voters in such situations—might cause perverse behavior that, in the absence of an adverse selection problem, would seem to indicate a failure of voter rationality.

Whether and how one can use elections and related institutions to mitigate or alleviate moral hazard is a classic concern (e.g., Barro (1973), Ferrejohn (1986), Austen-Smith and Banks (1989), Seabright (1996), Persson, Roland and Tabellini (1997), Shi and Svensson (2006), Bueno de Mesquita (2007), Fearon (2011)), but not our focus in this article. Classic moral hazard provides no \textit{ex post} incentive to the voter to replace the incumbent even if he or she “shirked” and did not exert a given level of effort on the voter’s behalf: moral hazard is induced by the electoral institution itself and can generally not be alleviated through replacement of the incumbent, as the incumbent’s successor will face the same incentive to shirk following the election.\(^2\) Moti-

\(^1\)Of course, moral hazard does not require that incumbents have \textit{no} incentive to work on behalf of the voter. Rather, it describes a situation in the incumbent’s optimal effort level is lower than that which the voter would exert on his or her own behalf.

\(^2\)While, in theory, the incumbent’s moral hazard problem can be mitigated through the use of \textit{ex ante} commitment to \textit{ex post} punishment, this approach is quite fragile and depends on the credibility of the voter’s commitment to the \textit{ex post} retention decision. Fearon (1999) provides a lucid and seminal discussion of this point.
vated by the debate about voters’ abilities to enforce electoral accountability (rather than debates about whether and when elections are a good accountability mechanism (e.g., Ferejohn (1999), Maskin and Tirole (2004), Alesina and Tabellini (2007, 2008))), our focus in this article is more properly aimed at a situation in which accountability through replacement has a first-order value. In a sense, focusing on adverse selection allows the intuitive strategy of “throw the bums out” to have some bite. Our theory indicates the theoretical and empirical difficulties that strategic reelection-seeking incumbents cause even for the most capable voters who are tasked with gauging exactly which incumbents are “the bums.”

1 The Baseline Model

We consider a model in which an incumbent politician is one of two types, \( t \in T = \{0, 1\} \) (where \( t = 1 \) with probability \( \pi \in (0, 1) \)), and this information is private information to the incumbent. The incumbent’s type denotes whether he or she has policy goals that (partially) conflict with those of the voter. This is the case when the type is \( t = 1 \); if the incumbent’s type is \( t = 0 \), then the incumbent’s policy goals are identical to those of the voter. In other words, any distortion of the behavior of an incumbent with type \( t = 0 \) is entirely due to his or her desire to be reelected. Accordingly, it is fair to say that any distortions of a type \( t = 0 \) incumbent from the voter’s best interests are “caused by” elections.

Upon observing his or her type, the incumbent (privately) observes the state of nature, \( \omega \in \Omega = \{0, 1\} \), where \( \omega = 1 \) with probability \( q \in (0, 1) \). After observing the state of nature, the incumbent chooses a policy, denoted by \( x \in X = \{0, 1\} \). We describe \( x \) as the level of prevention implemented by the incumbent. After \( x \) is chosen, an outcome, \( y \in Y = \{0, 1\} \), is realized. The outcome \( y = 1 \) represents a disaster occurring. The probability that a disaster
occurs ($y = 1$), conditional on $x$ and $\omega$, is denoted by $p(x, \omega) \in (0, 1)$.\footnote{We do not allow for a disaster to be either certain ($p_{x, \omega} = 1$) or impossible ($p_{x, \omega} = 0$) for uninteresting technical reasons. In particular, allowing for such cases introduces exogenously zero-probability histories (i.e., “paths of play”). Such possibilities simply provide us with more degrees of freedom to make our point and extra notation to carry around without adding to the substantive insight. Accordingly, we rule them out.}

Following the realization of $y$ (i.e., after the disaster has either happened or not), the incumbent chooses whether to pursue relief programs or not, denoted by $z \in \{0, 1\}$ with $z = 1$ representing the delivery of relief and $z = 0$ representing a decision to forego relief.

After $y$ is realized, a voter, $V$, observes $\hat{y}$ and then decides whether to reelect the incumbent or replace him or her with a challenger whose type, $t_C \in \{0, 1\}$, is independently drawn with the probability that $t_C = 1$ is $\pi_C$. The voter’s decision is denoted by $r = 1$ if he or she decides to reelect the incumbent and $r = 0$ otherwise.

The incumbent’s payoff function is

$$u_I(x, y, r, t) = tx - \alpha(y(1 - c_z z) + c_x x) + wr,$$

and the voter’s payoff function is

$$u_V(x, y, r, t, t_C, r) = -(y(1 - c_z z) + c_x x) - \phi(rt + (1 - r)t_C).$$

The parameters $\alpha > 0$, $w \geq 0$, $c_z \in [0, 1]$, $c_x \in [0, 1]$, and $\phi \geq 0$ are each exogenous and common knowledge. First consider the voter’s payoff function. The parameters $c_z$ represents the cost of prevention borne by the voter, $c_z$ represents the efficacy of relief spending, and $\phi$ represents the adverse selection problem faced by the voter. When $\phi = 0$, the voter does not consider the type of the challenger in his or her calculation of the (net) value of replacing the incumbent. When $\phi > 0$, the voter does consider this shadow of the future when making his or her reelection decision. We assume this particular specification of voter utility for disaster and relief spending because
it implies sensible preferences over these variables, as will become clear below.

In the incumbent’s payoff function, \( \alpha \) represents the “altruistic” motivation of the incumbent, which we assume is independent of the incumbent’s type. As \( \alpha \to 0 \), the incumbent becomes intrinsically indifferent to the voter’s welfare (i.e., the incumbent would only consider the voter’s welfare if induced to do so by the electoral process, as in canonical electoral agency models), and if \( \alpha = 1 \), the incumbent’s direct preferences mirror those of the voter with respect to disaster and relief spending.

However, even for \( \alpha = 1 \), the incumbent’s payoff differs from the voter in two important ways. First, the incumbent values holding office. This value is measured by \( w \): larger values of this parameter represent a stronger office-seeking motivation for the incumbent. As with \( \alpha \), we assume for simplicity that this motivation is independent of the incumbent’s type. Second, if \( t = 1 \), the incumbent values prevention spending directly, irrespective of its effect on the probability of a disaster. The capital projects involved in prevention give politicians opportunities for rent seeking such as personal profit from corrupt allocation of contracts or “vanity rents” from construction of elaborate projects even when they are not useful to the public. Type \( t = 1 \) politicians value these aspects of prevention; type \( t = 0 \) do not. This is why voters might care about politicians’ types.

The state of nature \( \omega \) affects the probability of a disaster \((y = 1)\) for a given prevention level \( x \). We assume that \( p(x, 0) < p(x, 1) \) for each \( x \in X \) and we focus on the case in which

\[
(p(0, 1) - p(1, 1))(1 - c_z) > c_x, \text{ and }
(p(0, 0) - p(1, 0))(1 - c_z) < c_x,
\]

so that prevention spending \((x = 1)\) is beneficial to the voter if and only if \( \omega = 1 \).

\(^4\)Note that we allow in principle for the possibility that \( p(0, 0) < p(1, 0) \), a case in which prevention spending actually increases the probability of a disaster when \( \omega = 0 \). This
The voter’s payoff function implies that the optimal complete information policy from the voter’s perspective, denoted by \((x^{**}(\omega), y^{**}(y))\) for each state of nature \(\omega\) and disaster occurrence \(y\), is
\[
x^{**}(\omega) = \omega, \quad \text{and} \\
z^{**}(y) = y.
\]
Simply put, the voter values prevention spending if and only if a disaster is sufficiently likely, and values relief spending if and only if a disaster actually occurs.

**Strategies and Beliefs.** A (pure) strategy for the incumbent consists of two functions. The first function is a mapping \(\sigma^x_I : \{0,1\} \times \{0,1\} \rightarrow [0,1]\) that selects a probability of setting prevention \(x = 1\) for each pair \((t, \omega)\). The second function is a mapping \(\sigma^z_I : (\{0,1\})^4 \rightarrow [0,1]\) that selects a probability of providing relief \((z = 1)\) for each quadruple \((t, \omega, x, y)\). When the context is clear, we write \(\sigma_I = (\sigma^x_I, \sigma^z_I)\) to denote the incumbent’s complete strategy.

A strategy for the voter is a mapping \(\sigma_V : (\{0,1\})^3 \rightarrow [0,1]\) that selects a probability of reelection \((i.e., r = 1)\) for each triple \((x, y, z)\). The voter’s beliefs are denoted by \(\beta : (\{0,1\})^3 \rightarrow [0,1]\) and designate, for each triple \((x, y, z)\), the voter’s subjective probability that \(t = 1\) upon observing \((x, y)\). In line with this, note that much of \(u_V\) is essentially a welfare benchmark.

In the game we analyze in this article, the voter has no direct control over \(x, y,\) or \(z\) (or, of course, \(t\) or \(t_C\)). Rather, a central point of this article is that elections are a coarse accountability mechanism \((e.g.,\text{, Fearon (1999)})\). Thus, the voter’s incentives when considering whether to reelect \((r = 1)\) or replace \((r = 0)\) the incumbent boil down to
\[
\hat{u}_V(r | x, y, z) = -\phi(r \beta(x, y, z) + (1 - r)\pi_C)
\]
possibility is relevant for equilibrium behavior when prevention spending is not directly observable, but the occurrence of the disaster is.
Thus, regardless of his or her type $t$, the incumbent prefers lower voter beliefs.

**Equilibrium.** Our equilibrium notion is perfect Bayesian equilibrium with beliefs satisfying the D1 refinement.\(^5\) An arbitrary equilibrium is denoted by $(\sigma_I^*, \sigma_V^*, \beta^*)$. Before continuing to the analysis, however, it is important to consider what exactly equilibrium “means” in this setting.

The foundation of a perfect Bayesian equilibrium in this setting is consistency between the voter’s inferences about the incumbent’s type and the incentives faced by each type of incumbent. That is, in equilibrium, beliefs and behavior are internally consistent. Accordingly, behavior as described by perfect Bayesian equilibrium is necessarily “rational.” Of course, equilibrium behavior within such a simple model may not be descriptively realistic. Indeed, we do not believe that the model utilized in this paper is a realistic model of elections: the electoral setting is very stark. We mention this because this is arguably the greatest value of the model: it is simple and illustrates the central logic of our argument. Put another way, this overly simple model of elections is sufficient to establish what one might call “possibility results.” Our argument in this article is that a model of completely rational (in fact, arguably “hyper-rational”) voters can produce inefficient outcomes and, indeed, be characterized by vote choices that appear pathological.

The basis for this conclusion in the setting we construct is based on information asymmetries. Accordingly, it is important to consider the role of our setting’s informational structure in obtaining our conclusions.

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\(^5\)This refinement requires that beliefs following any out-of-equilibrium path of play assign positive weight only to the incumbent type who would be “most likely” to benefit (out of equilibrium) from that deviation, relative to the expected payoff that type will receive from playing the equilibrium strategy. Informally, this refinement implies that “too little” prevention relative to the equilibrium path of play must lead the voter to infer that the incumbent is a faithful ($t = 0$) type, and “too much” prevention relative to the equilibrium path of player must lead the voter to infer that the incumbent is a biased ($t = 1$) type. It does not pin down beliefs following deviations with respect to relief spending, as both types have the same intrinsic preferences with respect to this spending.
1.1 The Informational Structure

Our framework, while purposely sparse, contains a key “wrinkle” from the standpoint of the informational structure. The efficient policy choice from the voters’ standpoint is completely determined by the realization of $\omega$: it determines the risk of a disaster, and disaster prevention spending is efficient if and only if this risk is high. However, the incumbent privately observes the state of nature $\omega$. That is, the voters can not explicitly condition the choice of policy on the facts on the ground because they can not observe them. This is a realistic assumption in most realms of policymaking: voters simply do not have the information required to make ex post efficient policy decisions.

This informational structure is key to our theoretical analysis, but it we believe it is also key from an empirical standpoint. Specifically, the existence of a real-world policymaking institution should be taken as somewhat indicative of the strategic environment characterizing the policies made by that institution. In other words, the fact that disaster policy is a public policy decision is itself informative about the challenges faced when attempting to make and implement choices about disaster prevention and relief. Pushing this point a little farther, the fact that disaster policy is “made” by elected politicians—as opposed to being left “to the market” or delegated to unelected/insulated bureaucrats or judges—is arguably itself a function of the nature of the technologies governing and preferences regarding disaster policy.

1.2 Equilibrium Analysis

The first result states a key feature of equilibrium voter behavior: if the voter cares about the future ($\phi > 0$) and believes, given the observed history of play $(x, y, z)$, that the incumbent is more likely to be a biased incumbent than the challenger is, the voter must replace the incumbent with the challenger. The converse holds as well: if the incumbent is deemed to be less likely to
Lemma 1 Suppose that $\phi > 0$. In equilibrium, if $\beta^*(x, y, z) > \pi_C$, then $\sigma^*_I(x, y, z) = 0$ and if $\beta^*(x, y, z) < \pi_C$, then $\sigma^*_I(x, y, z) = 1$.

Lemma 1 is both intuitive and powerful. In a nutshell, it implies that, if incumbents are sufficiently reelection-motivated, they will never separate in equilibrium. In other words, sufficiently strong career concerns on the part of the incumbents imply that both “good” and “bad” types of incumbents will behave the same in equilibrium. An implication of this is that, in such cases, the voter can not learn anything about the incumbent’s type from his or her choice of prevention policy. This is stated formally in the next proposition.\(^6\)

Proposition 1 Suppose that $w > 1 + \alpha(1 + c_x)$ and $\phi > 0$. In any PBE $(\sigma^*_I, \sigma^*_V, \beta^*)$ and for any triple $(x, y, z)$ reached with positive probability under $(\sigma^*_I, \sigma^*_V)$,

$$\beta^*(x, y, z) = \pi.$$ 

Proof: All proofs are contained in the appendix. 

Proposition 1, combined with the incumbent’s motivations when $t = 1$, implies that if, in an equilibrium strategy profile, (1) the incumbent is re-elected with positive probability after engaging in prevention spending and (2) prevention spending occurs with positive probability in equilibrium, then prevention spending must occur with certainty in the equilibrium. This is because, under the presumption that the reelection motivation, $w$, is large enough, the corrupt type of the incumbent ($t = 1$) will engage in prevention spending regardless of the state of nature ($\omega$) if the faithful type ($t = 0$) is willing to engage in prevention spending when the disaster risk is higher ($\omega = 1$). This fact, in conjunction with Proposition 1, implies that this type

\(^6\)Note that Proposition 1 does not utilize the D1 belief refinement, thereby strengthening the conclusion.
of behavior can be supported in equilibrium only if the faithful type also engages in prevention spending when it the disaster risk is low ($\omega = 0$), because otherwise the voter’s updated beliefs about the type of the incumbent would be biased toward the corrupt type after observing prevention spending (and would assign probability zero to the corrupt type after observing no prevention spending).

This logic suggests that there are essentially two possible perfect Bayesian equilibria when the incumbent is sufficiently reelection motivated: one in which the incumbent never engages in prevention spending and another in which the incumbent always engages in prevention spending. In fact, each of these types of behavior is supportable in equilibrium. The next proposition characterizes the perfect Bayesian equilibrium in which prevention spending never occurs—this perfect Bayesian equilibrium also satisfies the D1 refinement.

**Proposition 2** Suppose that $w > 1 + \alpha(1 - c_x - c_z)$ and $\phi > 0$. There is an equilibrium satisfying the D1 refinement in which prevention spending never occurs, regardless of either the incumbent’s type or the realized state of nature. Formally, the following strategy-belief profile, $(\sigma^*_I, \sigma^*_V, \beta^*)$, is a perfect Bayesian equilibrium with beliefs satisfying the D1 refinement:

\[
\begin{align*}
\sigma^*_I(t, \omega) &= 0 \text{ for all } (t, \omega) \in T \times \Omega, \\
\sigma^*_I(t, \omega, x, y) &= y \text{ for all } (t, \omega, x) \in T \times \Omega \times X, \\
\sigma^*_V(x, y, z) &= \begin{cases} 1 & \text{if } x = 0 \text{ and } y = z, \\
0 & \text{otherwise}, \end{cases} \\
\beta^*(x, y, z) &= \begin{cases} 1 & \text{if } x = 1, \\
\pi & \text{otherwise}. \end{cases}
\end{align*}
\]

Note that the conditions in Proposition 2 do not restrict $p$ or $c_x$. Thus, in this equilibrium, the incumbent’s choice of prevention spending is invariant
to the efficiency of prevention spending. Accordingly, the incumbent’s career concerns lead to inefficient prevention spending, precisely because the voter is rational and attempting to ferret out “bad types” of incumbents.\footnote{The equilibrium described in Proposition 2 is not unique (even among PBE with beliefs satisfying the D1 refinement). Nonetheless, as we show in the appendix, among equilibria with beliefs satisfying the D1 criteria, the nonuniqueness is with respect only to the choice of relief spending, \( z \).}

The next proposition demonstrates the importance of the D1 refinement on beliefs. Specifically, it presents an equilibrium in which, consistent with Proposition 1, the two incumbent types choose the same level of prevention spending. Unlike the equilibrium constructed in Proposition 2, however, both types of incumbents choose high prevention spending in this new equilibrium. This equilibrium does not satisfy the D1 refinement: the equilibrium behavior on the part of the good type is supported by perverse off-the-path beliefs that infer an incumbent who doesn’t engage in spending is the corrupt type. Thus, the D1 refinement rules out an equilibrium in which all incumbents engage in prevention spending regardless of \( \omega \). As with the equilibrium constructed in Proposition 2, this equilibrium exists regardless of the efficiency of prevention spending, so that the prevention spending in this example is inefficient (as is the spending in the equilibrium constructed in Proposition 2, though the relative inefficiency of the two equilibria will in general differ).

**Proposition 3** Suppose that \( w > 1 + \alpha(1 - c_z - c_y) \) and \( \phi > 0 \). The following strategy-belief profile, \((\sigma^*_I, \sigma^*_V, \beta^*)\), is a perfect Bayesian equilibrium:

\[
\begin{align*}
\sigma^*_I(t, \omega) &= 1 \text{ for all } (t, \omega) \in T \times \Omega, \\
\sigma^*_I(t, \omega, x, y) &= y \text{ for all } (t, \omega, x) \in T \times \Omega \times X, \\
\sigma^*_V(x, y, z) &= \begin{cases} 1 & \text{if } x = 1, \\
0 & \text{otherwise,} \end{cases} \\
\beta^*(x, y, z) &= \begin{cases} 1 & \text{if } x = 0, \\
\pi & \text{otherwise.} \end{cases}
\end{align*}
\]
2 Conclusion

This paper develops a model of electoral accountability with adverse selection in the context of disaster prevention and relief. Voters are uncertain about the benefit politicians obtain from prevention projects as a form of rent seeking. In view of this uncertainty, voters must ask what information is conveyed about politicians’ types if they engage in the construction of disaster prevention projects. Although many equilibria are possible in our model, a natural restriction on off-equilibrium-path beliefs suggests that voters consider it “bad news” about the incumbent’s type if she does in fact pursue prevention policies. In turn, incumbents must ask what information it conveys to voters if they engage in prevention spending. Given the inference voters draw from prevention spending in equilibrium, incumbents that desire to hold office, regardless of their preference for rent seeking at voters’ expense, will forego disaster prevention projects. Thus, the natural equilibrium of our model involves relief spending in response to a disaster, but no prevention spending in advance of it, even when this is beneficial for voters.

The result is important because it suggests that voters’ empirical tendency to reward relief but not prevention spending by incumbents is not necessarily due to voter myopia or some other behavioral pathology. To be sure, elections induce suboptimal policy choices by incumbents in our model. But this is inherent in the limits of the election as an instrument of accountability, in the face of uncertainty about politicians’ motivations and interests. The attendant suboptimal policy is the best that voters can achieve from this institution given this information asymmetry. Understanding the reason for the suboptimal policy observed is important because it structures the questions scholars should ask about voter education and the benefits of institutional reform.
A Appendix

The following proposition demonstrates that the equilibrium constructed in Proposition 2 is essentially unique insofar as any other equilibrium differs only with respect to the choice of relief spending.

Proposition 4 Suppose that $w > 1 + \alpha(1 - c_x - c_z)$ and $\phi \geq 0$. If $(\sigma_1^*, \sigma_y^*, \beta^*)$ is a perfect Bayesian equilibrium with beliefs satisfying the D1 refinement, then

$$
\sigma_1^{x*}(t, \omega) = 0 \text{ for all } (t, \omega) \in T \times \Omega,
$$

$$
\beta^*(x, y, z) = \begin{cases} 
1 & \text{if } x = 1, \\
\pi & \text{otherwise.}
\end{cases}
$$

Proof: Suppose that $(\sigma_1^*, \sigma_y^*, \beta^*)$ is a perfect Bayesian equilibrium and $\beta^*$ satisfies the D1 refinement.

By Proposition 1, $\beta^*(x, y, z) = \pi$ for any $(x, y, z)$ on the equilibrium path. Thus, suppose that $\sigma_1^{x*}(t, \omega) > 0$ for some $(t, \omega) \in \{0,1\}^2$. Then

Given the structure of $\sigma_1^{x*}$, all information sets in which $x = 1$ are off-the-equilibrium path, and $\beta^*$ correctly assigns probability one to $t = 1$ in such information sets. \hfill \blacksquare

Proposition 1 Suppose that $w > 1 + \alpha(1 + c_x)$ and $\phi > 0$. In any PBE $(\sigma_1^*, \sigma_y^*, \beta^*)$ and for any triple $(x, y, z)$ reached with positive probability under $(\sigma_1^*, \sigma_y^*)$,

$$
\beta^*(x, y, z) = \pi.
$$

Proof: Let $(\sigma_1^*, \sigma_y^*, \beta^*)$ be a PBE and consider any $(x, y, z)$ reached with positive probability under $(\sigma_1^*, \sigma_y^*)$. For the purpose of obtaining a contradiction suppose, contrary to the hypothesis, that

$$
\beta^*(x, y, z) \neq \pi. \quad (2)
$$
Let \((x_0, y_0, z_0)\) be a triple reached with positive probability under \((\sigma_I^*, \sigma_V^*)\) such that

\[ \beta^*(x_0, y_0, z_0) > \pi \]

and let \((x_1, y_1, z_1)\) be a triple reached with positive probability under \((\sigma_I^*, \sigma_V^*)\) such that

\[ \beta^*(x_1, y_1, z_1) > \pi. \]

By the supposition that there exists a triple \((x, y, z)\) reached with positive probability under \((\sigma_I^*, \sigma_V^*)\) satisfying inequality (2), such triples \((x_0, y_0, z_0)\) and \((x_1, y_1, z_1)\) must exist under \((\sigma_I^*, \sigma_V^*)\). By Lemma 1, \(\sigma_V^*(x_0, y_0, z_0) = 0\) and \(\sigma_V^*(x_1, y_1, z_1) = 1\). Now consider the following possibilities:

1. \(x_0 = x_1, y_0 = y_1, z_0 = z_1\). The biased incumbent is not best responding when choosing \(z\) following \((1, \omega, x_0, y_0)\) for some \(\omega \in \Omega\). In particular, for some \(\omega \in \Omega\), setting \(\sigma'_{I}(1, \omega, x_0, y_0) = \sigma^z_{I}(0, \omega, x_0, y_0)\) will strictly increase the biased incumbent’s conditional expected payoff by at least \(1 - c_z > 0\) (it might increase it by \(1 + c_z\)). Thus, \(\sigma^z_{I}\) is not sequentially rational, given \(\beta^*\). This logic also implies that we can presume that \(z_0 = z_1\) for the remainder of the proof.

2. \(x_0 = x_1, y_0 \neq y_1, z_0 = z_1\). Because \(\omega\) is independent of \(t\) and \(y\) depends only on \(\omega\) and \(x\), \(\beta^*(x, y, z) = \beta^*(x, 1 - y, z)\) for any \((x, y, z)\) reached with positive probability on the equilibrium path of play. This implies that we can presume that \(y_0 = y_1\) (and, by Step 1 above, \(z_0 = z_1\)) for the remainder of the proof.

3. \(x_0 \neq x_1, y_0 = y_1, z_0 = z_1\). The biased incumbent is not best responding when choosing \(x = x_0\) following \((1, \omega)\) for some \(\omega \in \Omega\). In particular, Step 1 above, combined with the supposition that \((\sigma_I^*, \sigma_V^*, \beta^*)\) is a PBE implies that \(\sigma^z_{I}(1, \omega, x_1, y) = \sigma^z_{I}(0, \omega, x_1, y)\) for each \(y \in Y\), which implies that

\[ \beta^*(x_1, y, z) > \pi \]
for all \((x_1, y, z)\) reached with positive probability (of which there is at least one). Thus, setting \(\sigma^*_I(1, \omega) = \sigma^*_I(0, \omega)\) for all \(\omega \in \Omega\) implies that the biased incumbent’s conditional expected payoff for any \(\omega\) will increase by at least \(w - \alpha(1 + c_x) - 1\). Because we have supposed that \(w > 1 + \alpha(1 + c_x)\), it follows that \(\sigma^*_I\) is not sequentially rational, given \(\beta^*\).

Because we can consider triples \((x_0, y, z)\) and \((x_1, y, z)\), it now follows that presuming that inequality (2) is satisfied by any triple \((x, y, z)\) reached with positive probability under \((\sigma^*_I, \sigma^*_V)\) implies that \(\sigma^*_I\) is not sequentially rational with respect to \(\beta^*\), contradicting the supposition that \((\sigma^*_I, \sigma^*_V, \beta^*)\) is a PBE. Thus, for any PBE \((\sigma^*_I, \sigma^*_V, \beta^*)\) and for any triple \((x, y, z)\) reached with positive probability under \((\sigma^*_I, \sigma^*_V)\), it must be the case that

\[
\beta^*(x, y, z) = \pi,
\]

as was to be shown.

\begin{proof}
We first verify the claim that \((\sigma^*_I, \sigma^*_V, \beta^*)\) is a PBE and then verify that \(\beta^*\) satisfies the D1 refinement.

Proposition 2. Suppose that \(w > 1 + \alpha(1 - c_z - c_x)\) and \(\phi > 0\). The following strategy-belief profile, \((\sigma^*_I, \sigma^*_V, \beta^*)\), is a perfect Bayesian equilibrium with beliefs satisfying the D1 refinement:

\[
\begin{align*}
\sigma^*_I(t, \omega) &= 0 \text{ for all } (t, \omega) \in T \times \Omega, \\
\sigma^*_I(t, \omega, x, y) &= y \text{ for all } (t, \omega, x) \in T \times \Omega \times X, \\
\sigma^*_V(x, y, z) &= \begin{cases} 
1 & \text{if } x = 0 \text{ and } y = z, \\
0 & \text{otherwise}, 
\end{cases} \\
\beta^*(x, y, z) &= \begin{cases}
1 & \text{if } x = 1, \\
\pi & \text{otherwise.}
\end{cases}
\end{align*}
\]

Proof: We first verify the claim that \((\sigma^*_I, \sigma^*_V, \beta^*)\) is a PBE and then verify that \(\beta^*\) satisfies the D1 refinement.
$(\sigma^*_I, \sigma^*_V, \beta^*)$ is a PBE. We first verify that the players’ strategies are sequentially rational, given $\beta^*$, and then demonstrate the consistency of $\beta^*$ with these strategies. To see that $\sigma^*_I$ is sequentially rational, note first that deviating from $\sigma^*_I$ results in a payoff loss of $w$ for all $(t, \omega, y) \in \{0, 1\}^3$ if $x = 0$. If $x = 1$, this deviation results in no change to the incumbent’s payoff, as this is off-the-equilibrium path.

Considering $\sigma^*_V$, note that choosing $x = 1$ after either $\omega$ will result in a payoff of at most $1 - \alpha c_x$, versus a worst case payoff (given $\sigma^*_I$) of $w - \alpha(1 - c_z)$ after choosing $x = 0$. Thus,

$$w > 1 + \alpha(1 - c_z - c_x) \Rightarrow w - \alpha(1 - c_z) > 1 - \alpha c_x,$$

so any deviation from $\sigma^*_I$ results in a strictly lower payoff.

The voter’s strategy, $\sigma^*_V$, given $\beta^*$, is clearly a best response: on the equilibrium path, the voter is indifferent between replacing and reelecting the incumbent. Off the equilibrium path, the voter is similarly indifferent unless $x = 1$. In this case, given $\beta^*$, the voter’s best response is to replace the incumbent.

Note that $\beta^*$ is correct on the equilibrium path of play, so that $(\sigma^*_I, \sigma^*_V, \beta^*)$ is a PBE.

$\beta^*$ satisfies the D1 refinement. The type $t = 1$ incumbent clearly has greater incentive to deviate with respect to $x$, given $\sigma^*_V$, and both types of incumbent have equal incentive to deviate with respect to the choice of $z$. Thus, $\beta^*$ satisfies the D1 refinement.
Proposition 3 Suppose that \( w > 1 + \alpha(1 - c_z - c_x) \) and \( \phi > 0 \). The following strategy-belief profile, \((\sigma^*_I, \sigma^*_V, \beta^*)\), is a perfect Bayesian equilibrium:

\[
\begin{align*}
\sigma^*_I(t, \omega) &= 1 \text{ for all } (t, \omega) \in T \times \Omega, \\
\sigma^*_I(t, \omega, x, y) &= y \text{ for all } (t, \omega, x) \in T \times \Omega \times X, \\
\sigma^*_V(x, y, z) &= \begin{cases} 1 & \text{if } x = 1, \\
0 & \text{otherwise}, \end{cases} \\
\beta^*(x, y, z) &= \begin{cases} 1 & \text{if } x = 0, \\
\pi & \text{otherwise.} \end{cases}
\end{align*}
\]

Proof: Demonstrating that \((\sigma^*, \beta^*)\) is a perfect Bayesian equilibrium is straightforward, mirroring the argument for Proposition 2, and therefore omitted. Instead, we demonstrate that the beliefs in this equilibrium, \(\beta^*\), do not satisfy the D1 refinement.

Let \(U^*(t)\) denote the equilibrium expected payoff of an incumbent of type \(t\) and

\[
D(x, t, \omega) = \{r : p(x, \omega)u_I(x, 1, r, t) + (1 - p(x, \omega))u_I(x, 0, r, t) \geq U^*(t)\}
\]

denote the set of responses by the voter that yield an incumbent of type \(t\) a weakly higher payoff than his or her equilibrium expected payoff, conditional on the state of nature, \(\omega\). Given the equilibrium strategies, \(\sigma^*\),

\[
\begin{align*}
D(0, 0, 0) &= \{1\}, \\
D(0, 0, 1) &= \{\}, \\
D(0, 1, 0) &= \{\}, \\
D(0, 1, 1) &= \{\}.
\end{align*}
\]

Thus, if the voter observes \(x = 0\), the D1 refinement requires that \(\beta\) assign zero probability to \(t = 1\), because such an incumbent can never do at least
as well from choosing such prevention spending and do as well as he or she can do by following his or her equilibrium strategy. Accordingly, $\beta^*$ does not satisfy the D1 refinement.

References


